GRAVITATIONAL WORMHOLES

4/29/2026

Presenter: Jiayue Yang

Universe **2024**, *10*(6), 257; https://doi.org/10.3390/universe10060257

Collaborate with Robert Mann and Mengqi Lu

UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

PI

Photo credit: @bruce.digital

Outline

- Introduction
- Wormhole series solutions
- Matching the series solutions
- Conclusions

INTRODUCTION

Background Information

- ❑ A wormhole is a tunnel-like structure that connects two distant regions of spacetime or even two separate universes.

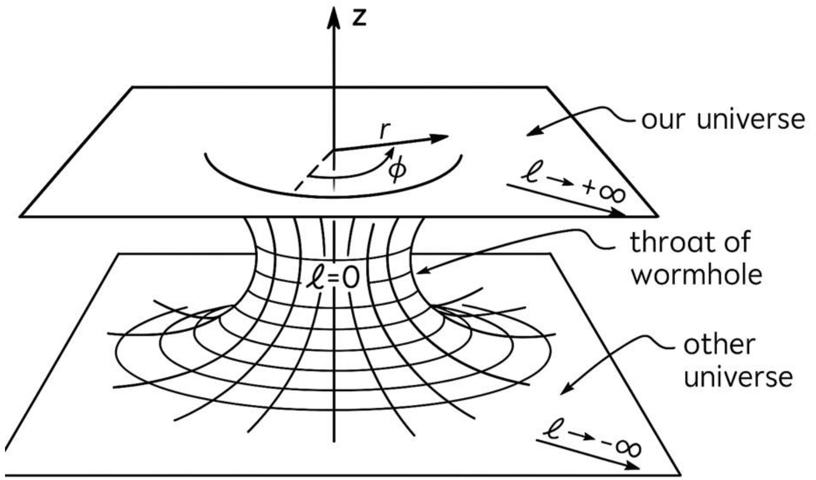
- ❑ Wormholes were first introduced as a traversable model for interstellar travel by Morris and Thorne in 1988.

Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity

Michael S. Morris; Kip S. Thorne

Check for updates

Am. J. Phys. 56, 395–412 (1988)
https://doi.org/10.1119/1.15620



Background Information

The Morris–Thorne wormhole spacetime is described by:

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- $\Phi(r)$: Redshift function, must be finite everywhere to avoid horizons
- $b(r)$: Shape function, determines the spatial geometry of the wormhole

  Throat occurs at minimum radius, i.e., $r = b(r)$
- ℓ: Radial proper distance from throat satisfies $\frac{d\ell}{dr} = \pm\left(1 - \frac{b(r)}{r}\right)^{-1/2}$
- Asymptotic flatness: $b(r)/r \to 0$, $\Phi(r) \to 0$ as $r \to \infty$

Motivation

- In general relativity, traversable wormholes require exotic matter (matter with negative energy density that violates energy conditions) to prevent the throat from collapsing.

- Open question: Can pure gravity (without any matter) support wormholes? (Gravitational wormholes: wormholes that only need pure gravity to construct.)

- Conjecture: Higher-curvature gravity theories (without any matter), such as Einsteinian Cubic Gravity, may allow for purely gravitational wormhole solutions.

- Set-up: We work in four-dimensional Einsteinian Cubic Gravity (ECG) that extends general relativity with cubic terms in curvature.

Set-up

The action of 4-dimensional Einsteinian Cubic Gravity is [31-35]

$$\mathcal{I} = \frac{1}{16\pi}\int d^4x\sqrt{-g}(-2\Lambda_0 + R + \alpha\mathcal{P} + \beta\mathcal{C} + \gamma\mathcal{C}'),$$

where α, β and γ are coupling constants, P, C and C′ are cubic densities given by

$$\mathcal{P} = 12R_{a\ b}^{\ c\ d}R_{c\ d}^{\ e\ f}R_{e\ f}^{\ a\ b} + R_{ab}^{\ \ cd}R_{cd}^{\ \ ef}R_{cd}^{\ \ ab} - 12R_{abcd}R^{ac}R^{bd} + 8R_a^bR_b^cR_c^a,$$

$$\mathcal{C} = \frac{1}{2}RR_a^bR_b^a - 2R^{ac}R^{bd}R_{abcd} - \frac{1}{4}RR_{abcd}R^{abcd} + R^{de}R_{abcd}R^{abc}{}_e,$$

$$\mathcal{C}' = R_a^bR_b^cR_c^a - \frac{3}{4}RR_a^bR_b^a + \frac{1}{8}R^3.$$

UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

Set-up

The field equations is given by

$$\mathcal{E}^{a}_{\ b} := \frac{g^{ac}}{\sqrt{-g}} \frac{\delta \mathcal{I}}{\delta g^{cb}} = T^{a}_{\ b}$$

where T_{ab} is the matter stress-energy tensor, and we shall let it be zero.

In the general static spherically symmetric (GSSS) ansatz

$$ds^2 = -f(r)dt^2 + \frac{1}{N(r)f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

the field equations depend on α and 2β + γ because the densities C and C′ are linearly dependent. We shall henceforth set γ = 0 without loss of generality.

Set-up

The two independent field equation

$$
\begin{aligned}
&-\frac{1}{8r^5}\Big\{8r\Big(r^2(-1+r^2\Lambda)+6\alpha N^2 f'^2(-1+r^2 f'N')+rNf'(r^2+12\alpha f'N')\Big)\\
&+2f\Big[4r^2N'(r^2+15\alpha f'N')+rN\Big(3r^2(74\alpha+5\beta)f'^2N^2+4r(r+48\alpha N'f'')\\
&-24f'\Big((8\alpha-\beta)N'-8r\alpha N''\Big)\Big)+24r\alpha N^3\Big(4f'^2+r^2f''^2+rf'(-4f''+rf''')\Big)\\
&+12N^2\Big(f'\Big(8\alpha+r^3(22\alpha+\beta)N'f''\Big)+8r^2\alpha f'^2\Big(-3N'+rN''\Big)+4r\alpha\Big(-2f''+rf'''\Big)\Big)\Big]\\
&+6f^3\Big[r^2N'^2\Big(-4\alpha N'+r(8\alpha+\beta)N''\Big)+4N^2\Big(2\beta N'-r\Big((-4\alpha+\beta)N''+4r\alpha N'''\Big)\Big)\\
&+rN\Big((8\alpha-7\beta)N'^2+2r^2(8\alpha+\beta)N''^2+2rN'\Big(-20\alpha N''+r(8\alpha+\beta)N'''\Big)\Big)\Big]\\
&+3f^2\Big[rN'\Big(6\beta N'+r^2(28\alpha+3\beta)f'N'^2+16r\alpha N''\Big)-32\alpha N^3\Big(2f'+r(-2f''+rf''')\Big)\\
&+2N\Big(2r^2N'^2\Big(-52\alpha f'+r(26\alpha+3\beta)f''\Big)+N'\Big(-8\beta+r^3(108\alpha+11\beta)f'N''\Big)\\
&+4r\Big((-4\alpha+\beta)N''+4r\alpha N'''\Big)\Big)+4rN^2\Big[r\Big(r(12\alpha+\beta)f''N''+N'\Big(-48\alpha f''+r(8\alpha+\beta)f'''\Big)\Big)\\
&+f'\Big(6(8\alpha-\beta)N'+4r\alpha(-12N''+rN''')\Big)\Big]\Big]\Big\}=0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{8r^5}\Big\{-8r\Big(r^2(-1+r^2\Lambda)+6\alpha N^2 f'^2(-1+r^2 f'N')+rNf'(r^2+12\alpha f'N')\Big)\\
&+6\beta f^3N\Big(rN'^2+N(-8N'+4rN'')\Big)-2f\Big[-12r^2\alpha f'N^2+rN\Big(3r^2(26\alpha+3\beta)f'^2N^2\\
&+4r\Big(r+12\alpha N'f''\Big)+24f'\Big((-2\alpha+\beta)N'+2r\alpha N''\Big)\Big)+24r\alpha N^3\Big(4f'^2+r^2f''^2\\
&+rf'(-4f''+rf''')\Big)+12N^2\Big(f'\Big(8\alpha+r^3(12\alpha+\beta)N'f''\Big)+2r^2\alpha f'^2(-8N'+rN'')\\
&+4r\alpha(-2f''+rf''')\Big)\Big]+3f^2\Big[rN'^2\Big(2\beta+r^2(8\alpha+\beta)f'N'\Big)-2N\Big(2r^2N'^2\Big(-4\alpha f'\\
&+r(8\alpha+\beta)f''\Big)+4r\beta N''+N'\Big(-8\beta+3r^3(8\alpha+\beta)f'N''\Big)\Big)+32\alpha N^3\Big(2f'+r\\
&\Big(-2f''+rf'''\Big)\Big)-4rN^2\Big(2f'\Big((8\alpha-3\beta)N'-8r\alpha(N'')\Big)+r\Big(r(8\alpha+\beta)f''N''+N'\\
&\Big(-16\alpha f''+r(8\alpha+\beta)f'''\Big)\Big)\Big)\Big]\Big\}=0
\end{aligned}
$$

WORMHOLE SERIES SOLUTIONS

Series solutions

Now let us consider the following metric ansatz with the advantage that the domain of x is finite $x \in [0, 1)$

$$ds^2 = -\frac{r_0^2 g(x)}{(1-x)^2} d\tilde{t}^2 + \frac{r_0^2 dx^2}{n(x)g(x)(1-x)^2} + \frac{r_0^2}{(1-x)^2}(d\theta^2 + \sin^2\theta d\phi^2),$$

where the transformation is

$$x = 1 - r_0/r \qquad \tilde{t} = t/r_0 \qquad n = N \quad g = \frac{fr_0^2}{r^2}$$

For traversable wormhole solutions, the metric functions must be every-where positive,

with n=0 and g is finite positive at x = 0 (wormhole throat $r=r_0$), infinity compactified to x = 1.

Large-r solutions

We now consider asymptotically AdS wormholes ($n(1)=1$, $g(1)=a_0$), substitute the series solutions in the distant region at large r ($x=1$)

$$g = a_0 + \sum_{n=2}^{\infty} a_n (1-x)^n, \quad n = 1 + \sum_{n=3}^{\infty} b_n (1-x)^n, \quad a_0 = r_0^2/l^2 \equiv L r_0^2, \quad a_2 = 1 + \delta$$

into the field equations and solve them order by order. The lowest two orders of field equation yield

$$24\alpha L^2 + 1 = 0$$

Large-r solutions

The large-r series solutions (with β=0) are

$$g = a_0 + a_2(1-x)^2 + \frac{4}{5}a_0 b_3(1-x)^3 - \frac{333a_0^2 b_3^2}{200(a_2-1)}(1-x)^4 + \frac{3b_3\left(-400a_2 + 800a_2^2 - 400a_2^3 + 7923a_0^3 b_3^2\right)}{3500(a_2-1)^2}(1-x)^5 + \cdots ,$$

$$n = 1 + b_3(1-x)^3 - \frac{81a_0 b_3^2}{20(a_2-1)}(1-x)^4 + \frac{3b_3\left(-400a_2 + 800a_2^2 - 400a_2^3 + 8271a_0^3 b_3^2\right)}{1000a_0(a_2-1)^2}(1-x)^5 + \cdots ,$$

where we set β=0, and a_0, a_2 and b_3 are the only free variables in this solution.

Near throat solutions

Series solutions in near throat region at $r=r_0$ $(x=0)$ satisfy $n(0)=0$, $g(0) \neq 0$

$$n = \sum_{n=1}^{\infty} A_n x^n \qquad g = B_0 + \sum_{n=1}^{\infty} B_n x^n, \quad B_0 \neq 0,$$

Plug these into field equations, we get the near throat series solutions (with $\beta=0$) , whose coefficients are fully determined by A_1, B_0 and a_0

$$n_{th} = A_1 x - \frac{72\left(A_1 B_0 + \frac{4}{3}\right) a_0^3 - 8\left(A_1^2 B_0^2 - \frac{7}{2} A_1 B_0 - 6\right) a_0^2 - 5A_1^4 B_0^4 - 9A_1^3 B_0^3 - 4A_1^2 B_0^2}{4\left(A_1 B_0 + 1\right)^2 A_1 B_0^2} x^2 + \cdots$$

$$g_{th} = B_0 + 2\frac{8a_0^3 + 4a_0^2 - A_1^3 B_0^3 - B_0^2 A_1^2}{A_1^2 B_0 \left(A_1 B_0 + 1\right)} x + \cdots$$

MATCHING THE SERIES SOLUTIONS

Matching the Solutions

We expect the far-region series matches with the near-throat series as long as a wormhole solution exists for some values of a_0, a_2, b_3, A_1, B_0. We do this by minimizing the objective function below

$$\Delta \equiv (\Delta g)^2 + (\Delta g')^2 + (\Delta n)^2 + (\Delta n')^2$$

as a function of the parameters (a_0, a_2, b_3, A_1, B_0), where $\Delta F \equiv F_\infty - F_{th}$

Wormhole boundary condition

- Support a throat: n(0)=0, g(0) ≠ 0
- Asymptotically AdS: n(1)=1, g(1)=a_0

Wormhole examples (g(x) solid, n(x) dashed)

- Example 1

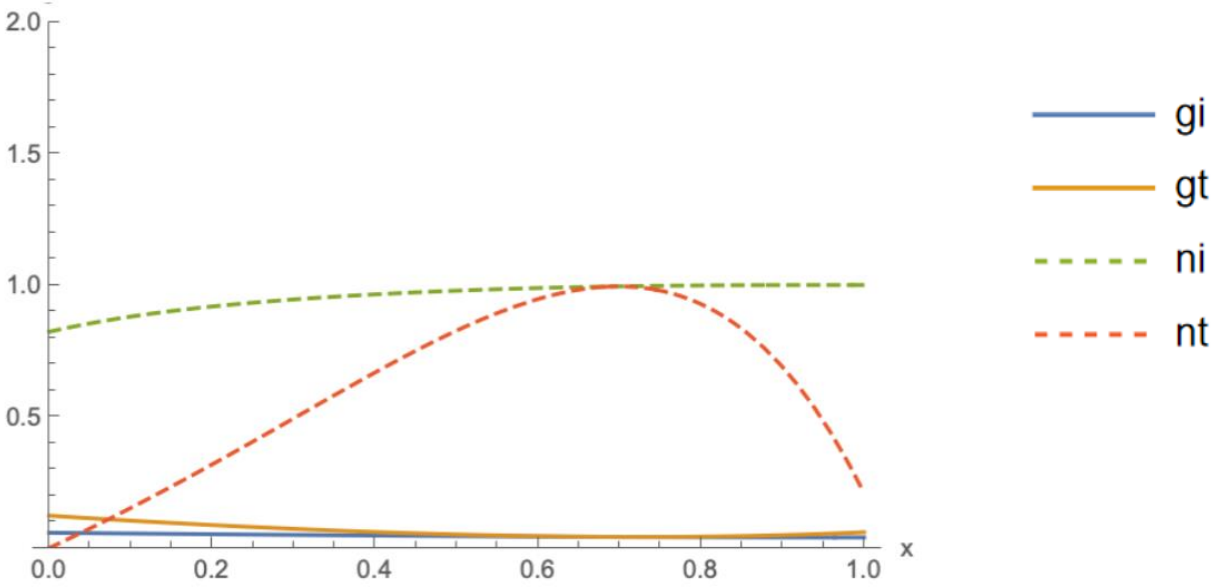


Figure 1. Plots of the series solutions $[g(x), n(x)]$ for both large-r [(solid blue, dashed green] and near-throat [(solid orange, dashed red]. The solutions smoothly match at $x = 0.694284$ for the parameters $a_2 = 0.0238388$, $b_3 = -0.2000$ $A_1 = 1.44815$, $B_0 = 0.122817$, and $a_0 = 0.04000$.

Example 2

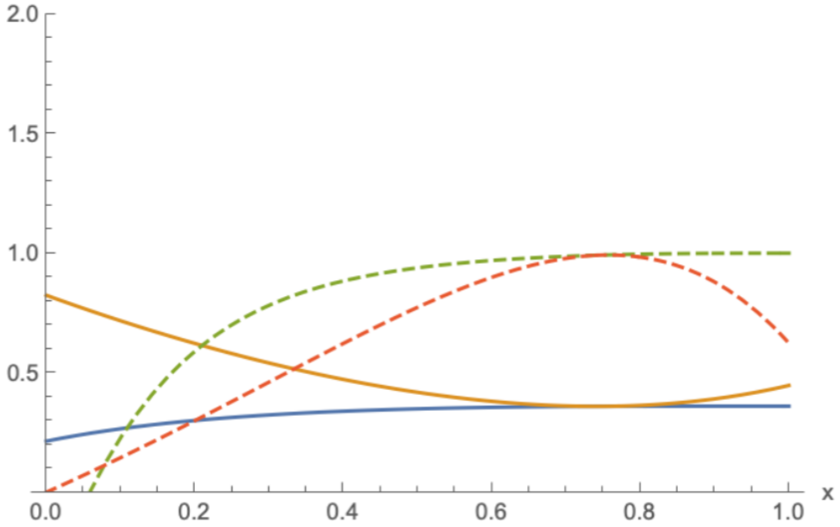


Figure 2. Plots of the series solutions $[g(x), n(x)]$ for both large-r [(solid blue, dashed green] and near-throat [(solid orange, dashed red]. The solutions smoothly match at $x = 0.694284$ for the parameters $a_2 = 0.029988289977868732$, $b_3 = -0.6000$ $A_1 = 1.3906651114275435$, $B_0 = 0.8239700690981224$, and $a_0 = 0.36000$.

Wormhole examples with beta

What if we want to minimize the objective function below to match the 2nd derivative?

$$\Delta(x_0) \equiv \left(\frac{\Delta g}{0!}\right)^2 + \left(\frac{\Delta g'}{1!}\right)^2 + \left(\frac{\Delta g''}{2!}\right)^2 + \left(\frac{\Delta n}{0!}\right)^2 + \left(\frac{\Delta n'}{1!}\right)^2 + \left(\frac{\Delta n''}{2!}\right)^2$$

We need to introduce a new parameter to dial!

$$(\beta, r_0, a_2, b_3, A_1, B_0)$$

Wormhole examples with beta

If beta is non-zero, we can still get the far-region series solutions

$$g = Lr_0^2 + a_2(1-x)^2 - \frac{2}{5}b_3 L r_0^2 (1-x)^3 \left(9\beta L^2 - 2\right) - \frac{9 b_3^2 L^2 r_0^4 (1-x)^4 \left(828\beta^2 L^4 - 363\beta L^2 + 37\right)}{200(a_2-1)}$$

$$n = 1 + b_3(1-x)^3 + \frac{27 b_3^2 L r_0^2 (1-x)^4 \left(11\beta L^2 - 3\right)}{20(a_2-1)} + \frac{3 b_3 (1-x)^5 \left(-400 a_2^3 + 800 a_2^2 - 400 a_2 + 9 b_3^2 L^3 r_0^6 \left(11916\beta^2 L^4 - 6621\beta L^2 + 919\right)\right)}{1000(a_2-1)^2 L r_0^2}$$

Wormhole examples with beta

The near throat series solutions with beta are

$$n_{th} = A_1 x + \frac{1}{8A_1 B_0^2 \left(A_1 B_0 \left(3\beta L^2 - 1\right) - 1\right)^2}$$

$$\Big\{ 2A_1^2 B_0^2 \left(1 - 3\beta L^2\right)\left(A_1^2 B_0^2 \left(5 - 12\beta L^2\right) + 9A_1 B_0 + 4\right)$$

$$-8L^2 {r_0}^4 \left(2A_1^2 B_0^2 \left(3\beta L^2 - 1\right) + A_1 B_0 \left(7 - 24\beta L^2\right) + 12\right)$$

$$+48 L^3 {r_0}^6 \left(A_1 B_0 \left(8\beta L^2 - 3\right) - 4\right) \Big\} x^2 + \cdots$$

$$g_{th} = B_0 - \frac{2L^2 \left(3A_1^2 B_0^2 (A_1 B_0 + 1)\left(\beta - \frac{1}{3L^2}\right) + 8L{r_0}^6 + 4{r_0}^4\right)}{A_1^2 B_0 \left(A_1 B_0 \left(3\beta L^2 - 1\right) - 1\right)} x + \cdots$$

Wormhole examples with beta

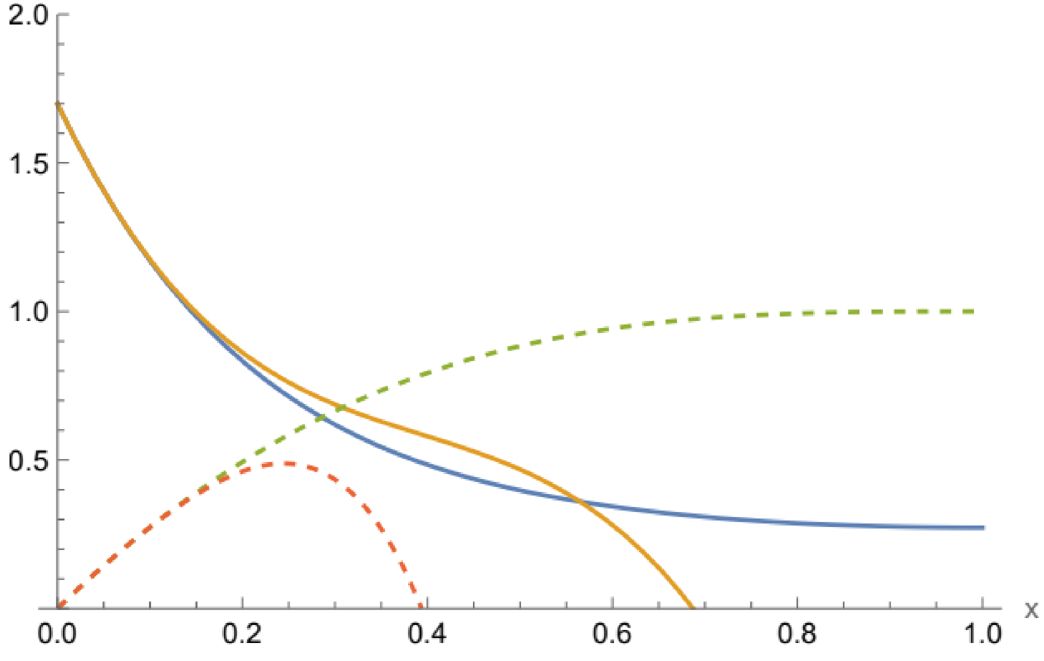
- 

FIG. 1. Plots of the series solutions $[g(x), n(x)]$ for both large-r [(solid blue, dashed green] and near-throat [(solid orange, dashed red]. The solutions smoothly match with $\Delta = 1.03924 \times 10^{-19}$ at $x = x_0 = 0.00505304$ for the parameters $\beta = 0.505553, a_2 = 0.318022, b_3 = -0.680002, r_0 = 0.522375, A_1 = 3.03285, B_0 = 1.69888$.

- With beta we can matches the second derivative

Wormhole examples with beta

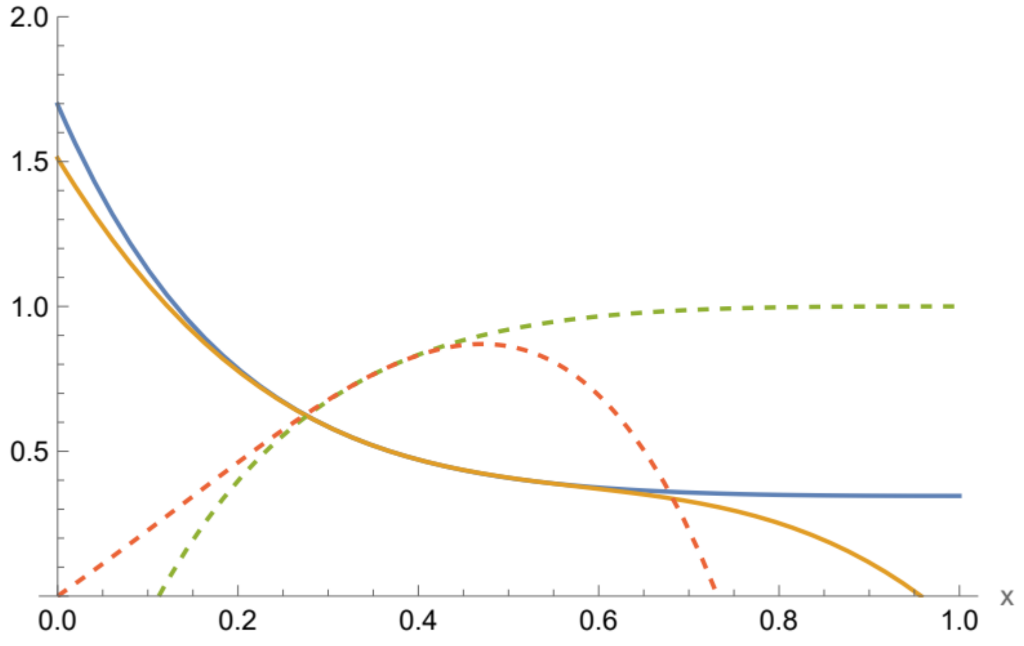
- 

FIG. 2. Plots of the series solutions $[g(x), n(x)]$ for both large-r [(solid blue, dashed green] and near-throat [(solid orange, dashed red]. The solutions smoothly match with $\Delta = 3.22544 \times 10^{-15}$ at $x = x_0 = 0.35156$ for the parameters $\beta = 0.792546, a_2 = 0.0368723, b_3 = -0.327935, r_0 = 0.588074, A_1 = 2.1819, B_0 = 1.51153$.

- With beta we can matches the second derivative

UNIVERSITY OF WATERLOO | FACULTY OF SCIENCE

Wormhole examples with beta

▪

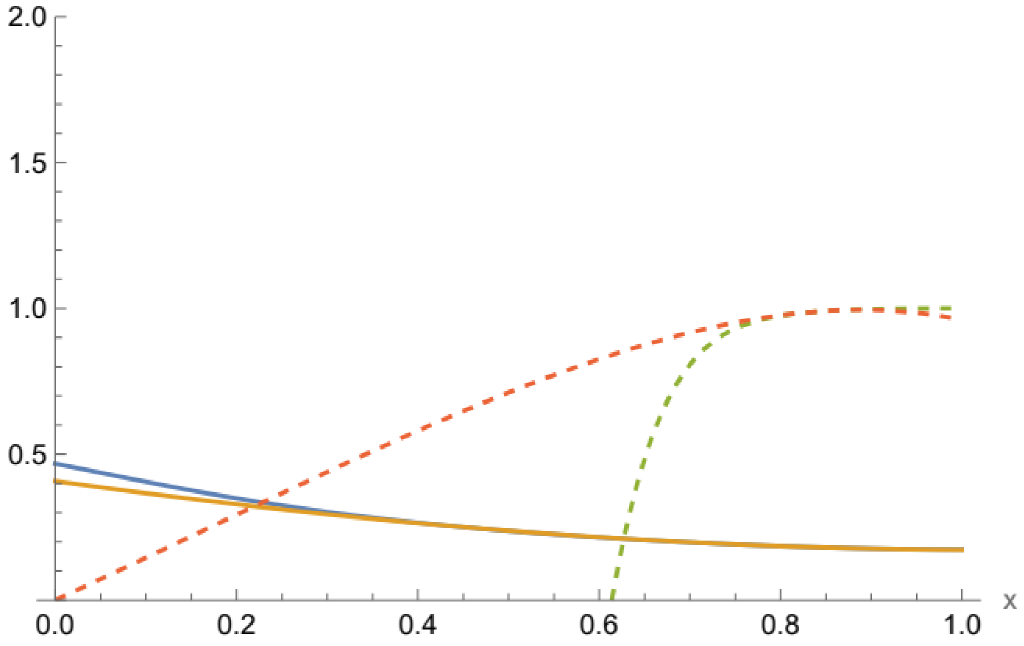


FIG. 3. Plots of the series solutions $[g(x), n(x)]$ for both large-r [(solid blue, dashed green] and near-throat [(solid orange, dashed red]. The solutions smoothly match with $\Delta = 3.2155 \times 10^{-16}$ at $x = x_0 = 0.83693$ for the parameters $\beta = 0.0706116, a_2 = 0.34649, b_3 = -3.53033, r_0 = 0.41619, A_1 = 1.43351, B_0 = 0.408057$.

- With beta we can matches the second derivative

CONCLUSIONS

Conclusions

- ✓We solve Einsteinian Cubic gravity field equations to get far-region series solutions and near throat series solutions
- ✓We require the two solutions to be connected smoothly, to get the parameters in the coefficients in the series solution.
- ✓We also investigate the case when beta is non-zero, and match to series solutions to the second order derivatives.
- ✓Results: Einsteinian Cubic gravity contains wormhole solutions that are purely gravitational.

References

[1] M.S. Morris, K.S. Thorne and U. Yurtsever, Wormholes, Time Machines, and the Weak Energy Condition, Phys. Rev. Lett. **61** (1988) 1446.

[2] M.S. Morris and K.S. Thorne, Wormholes in space-time and their use for interstellar travel: A tool for teaching general relativity, Am. J. Phys. **56** (1988) 395.

[3] M. Visser, Lorentzian wormholes: From Einstein to Hawking, Woodbury (1995).

[4] M. Visser, From wormhole to time machine: Comments on Hawking's chronology protection conjecture, Phys. Rev. D **47** (1993) 554 [hep-th/9202090].

[5] L.H. Ford and T.A. Roman, Quantum field theory constrains traversable wormhole geometries, Phys. Rev. D **53** (1996) 5496 [gr-qc/9510071].

[6] T. Harko, F.S.N. Lobo, M.K. Mak and S.V. Sushkov, Modified-gravity wormholes without exotic matter, Phys. Rev. D **87** (2013) 067504 [1301.6878].

[7] K.K. Nandi, A. Islam and J. Evans, Brans wormholes, Phys. Rev. D **55** (1997) 2497 [0906.0436].

[8] X. Yue and S. Gao, Stability of Brans-Dicke thin shell wormholes, Phys. Lett. A **375** (2011) 2193 [1105.4310].

[9] F.S.N. Lobo and M.A. Oliveira, General class of vacuum Brans-Dicke wormholes, Phys. Rev. D **81** (2010) 067501 [1001.0995].

[10] S.V. Sushkov and S.M. Kozyrev, Composite vacuum Brans-Dicke wormholes, Phys. Rev. D **84** (2011) 124026 [1109.2273].

[11] F.S.N. Lobo and M.A. Oliveira, Wormhole geometries in f(R) modified theories of gravity, Phys. Rev. D **80** (2009) 104012 [0909.5539].

[12] N.M. Garcia and F.S.N. Lobo, Wormhole geometries supported by a nonminimal curvature-matter coupling, Phys. Rev. D **82** (2010) 104018 [1007.3040].

[13] N. Montelongo Garcia and F.S.N. Lobo, Nonminimal curvature-matter coupled wormholes with matter satisfying the null energy condition, Class. Quant. Grav. **28** (2011) 085018 [1012.2443].

[14] A. Anabalon, J. Oliva and C. Quijada, Fully resonant scalars on asymptotically AdS wormholes, Phys. Rev. D **99** (2019) 104022 [1903.08239].

[15] D. Lovelock, Divergence-free tensorial concomitants, aequationes mathematicae **4** (1970) 127.

[16] D. Lovelock, The Einstein tensor and its generalizations, J. Math. Phys. **12** (1971) 498.

[17] B. Bhawal and S. Kar, Lorentzian wormholes in Einstein-Gauss-Bonnet theory, Phys. Rev. D **46** (1992) 2464.

[18] W. Wang and X.-Y. Li, Wormhole solution in D-dimensional Einstein and Lovelock theories, Nuovo Cim. B **111** (1996) 1101.

[19] Y.-W. Shang and J.-J. Xu, Wormhole solution in Lovelock gravity theory, Chin. Phys. Lett. **16** (1999) 85.

[20] H. Maeda and M. Nozawa, Static and symmetric wormholes respecting energy conditions in Einstein-Gauss-Bonnet gravity, Phys. Rev. D **78** (2008) 024005 [0803.1704].

[21] M.H. Dehghani and Z. Dayyani, Lorentzian wormholes in Lovelock gravity, Phys. Rev. D **79** (2009) 064010 [0903.4262].

[22] M.R. Mehdizadeh and F.S.N. Lobo, Novel third-order Lovelock wormhole solutions, Phys. Rev. D **93** (2016) 124014 [1604.02913].

[23] M.R. Mehdizadeh and N. Riazi, Cosmological wormholes in Lovelock gravity, Phys. Rev. D **85** (2012) 124022.

[24] M.R. Mehdizadeh, M. Kord Zangeneh and F.S.N. Lobo, Einstein-Gauss-Bonnet traversable wormholes satisfying the weak energy condition, Phys. Rev. D **91** (2015) 084004 [1501.04773].

References

[25] J. Oliva and S. Ray, A new cubic theory of gravity in five dimensions: Black hole, Birkhoff's theorem and C-function, Class. Quant. Grav. **27** (2010) 225002 [1003.4773].

[26] R.C. Myers and B. Robinson, Black Holes in Quasi-topological Gravity, JHEP **08** (2010) 067 [1003.5357].

[27] J. Oliva and S. Ray, Birkhoff's Theorem in Higher Derivative Theories of Gravity, Class. Quant. Grav. **28** (2011) 175007 [1104.1205].

[28] J. Oliva and S. Ray, Birkhoff's Theorem in Higher Derivative Theories of Gravity II, Phys. Rev. **D86** (2012) 084014 [1201.5601].

[29] M.H. Dehghani, A. Bazrafshan, R.B. Mann, M.R. Mehdizadeh, M. Ghanaatian and M.H. Vahidinia, Black Holes in Quartic Quasitopological Gravity, Phys. Rev. **D85** (2012) 104009 [1109.4708].

[30] A. Cisterna, L. Guajardo, M. Hassaine and J. Oliva, Quintic quasi-topological gravity, JHEP **04** (2017) 066 [1702.04676].

[31] R.A. Hennigar, D. Kubizňák and R.B. Mann, Generalized quasitopological gravity, Phys. Rev. D **95** (2017) 104042 [1703.01631].

[32] P. Bueno and P.A. Cano, On black holes in higher-derivative gravities, Class. Quant. Grav. **34** (2017) 175008 [1703.04625].

[33] J. Ahmed, R.A. Hennigar, R.B. Mann and M. Mir, Quintessential Quartic Quasi-topological Quartet, JHEP **05** (2017) 134 [1703.11007].

[34] P. Bueno and P.A. Cano, Universal black hole stability in four dimensions, Phys. Rev. **D96** (2017) 024034 [1704.02967].

[35] P. Bueno, P.A. Cano and R.A. Hennigar, (Generalized) quasi-topological gravities at all orders, Class. Quant. Grav. **37** (2020) 015002 [1909.07983].

[36] R.A. Hennigar, Criticality for charged black branes, JHEP **09** (2017) 082 [1705.07094].

[37] M. Mir and R.B. Mann, On generalized quasi-topological cubic-quartic gravity: thermodynamics and holography, JHEP **07** (2019) 012 [1902.10906].

[38] M. Mir, R.A. Hennigar, J. Ahmed and R.B. Mann, Black hole chemistry and holography in generalized quasi-topological gravity, JHEP **08** (2019) 068 [1902.02005].

[39] Y.-Z. Li, H.-S. Liu and H. Lu, Quasi-Topological Ricci Polynomial Gravities, JHEP **02** (2018) 166 [1708.07198].

[40] Y.-Z. Li, H. Lu and J.-B. Wu, Causality and a-theorem Constraints on Ricci Polynomial and Riemann Cubic Gravities, Phys. Rev. **D97** (2018) 024023 [1711.03650].

[41] Y.-Z. Li, H. Lu and Z.-F. Mai, Universal Structure of Covariant Holographic Two-Point Functions In Massless Higher-Order Gravities, JHEP **10** (2018) 063 [1808.00494].

[42] P. Bueno, P.A. Cano, J. Moreno and A. Murcia, All higher-curvature gravities as Generalized quasi-topological gravities, JHEP **11** (2019) 062 [1906.00987].

[43] P. Bueno and P.A. Cano, Einsteinian cubic gravity, Phys. Rev. D **94** (2016) 104005 [1607.06463].

[44] M. Barriola and A. Vilenkin, Gravitational Field of a Global Monopole, Phys. Rev. Lett. **63** (1989) 341.

[45] X. Shi and X.-z. Li, The Gravitational field of a global monopole, Class. Quant. Grav. **8** (1991) 761 [0903.3085].

UNIVERSITY OF WATERLOO

FACULTY OF SCIENCE

Thank you for your attention!

APPENDIX

Wormhole examples

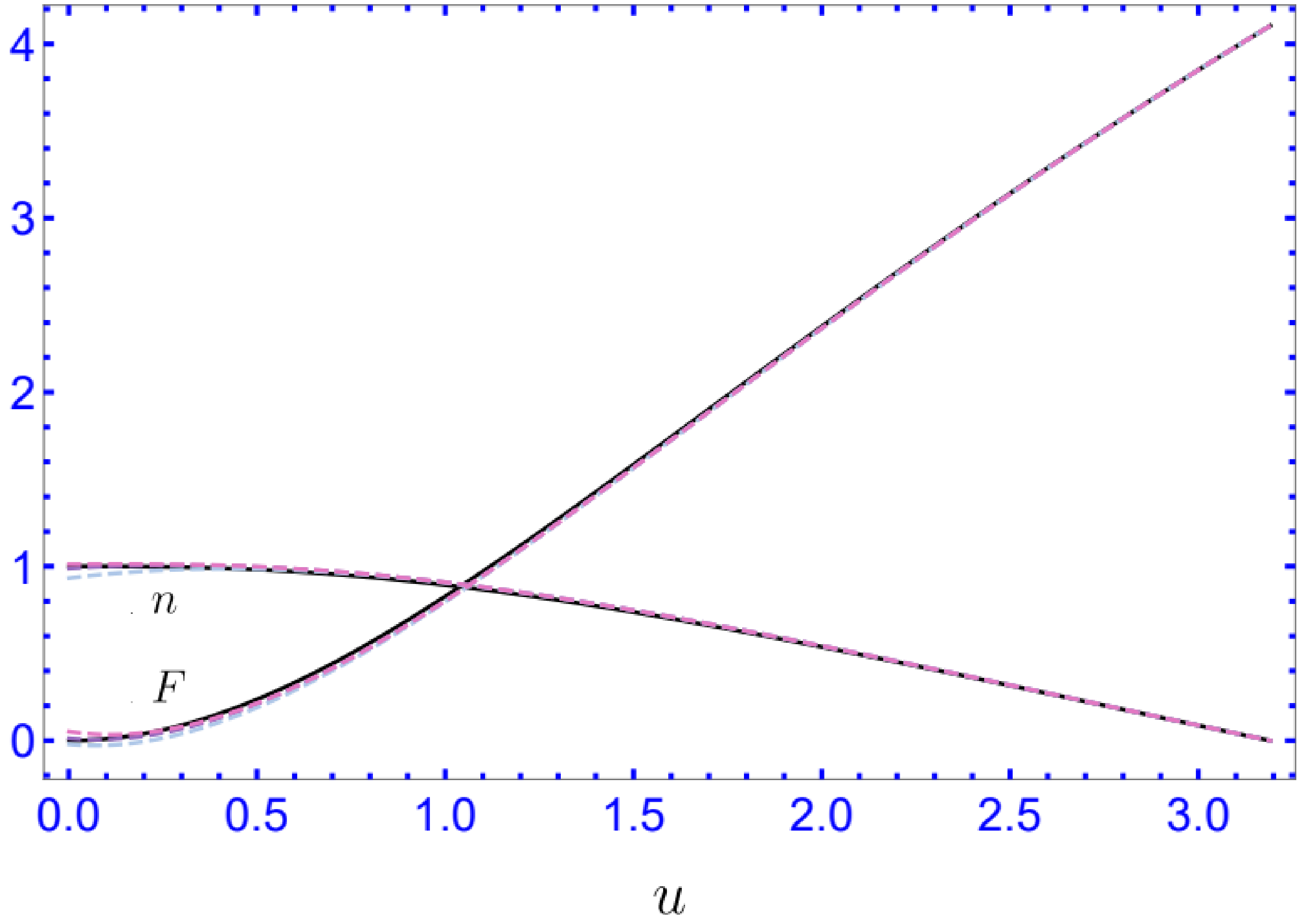
Our construction relies on the shooting method, namely, looking for initial conditions satisfying (1.6) at $r = \infty$ such that the wormhole boundary conditions are fulfilled at the throat r_{th}. For this purpose, we take $u \equiv 1/r$, $F(u) \equiv -1/g_{tt}$, $n(u) \equiv -g^{rr}/g_{tt}$ to rewrite the metric ansatz (1.5) as

$$ds^2 = -\frac{1}{F(u)}dt^2 + \frac{1}{u^4}\frac{F(u)}{n(u)}du^2 + \frac{1}{u^2}(d\theta^2 + \sin^2\theta d\phi^2). \tag{2.1}$$

$$F(u) = F_2 u^2 + F_4 u^4 + \sum_{i=5}^{\infty} F_i u^i, \quad n(u) = 1 + \sum_{i=3}^{\infty} n_i u^i,$$

$$F(u) = \sum_{i=0}^{\infty} \tilde{F}_i (u - u_{\text{th}})^i, \quad n(u) = \sum_{i=1}^{\infty} \tilde{n}_i (u - u_{\text{th}})^i,$$

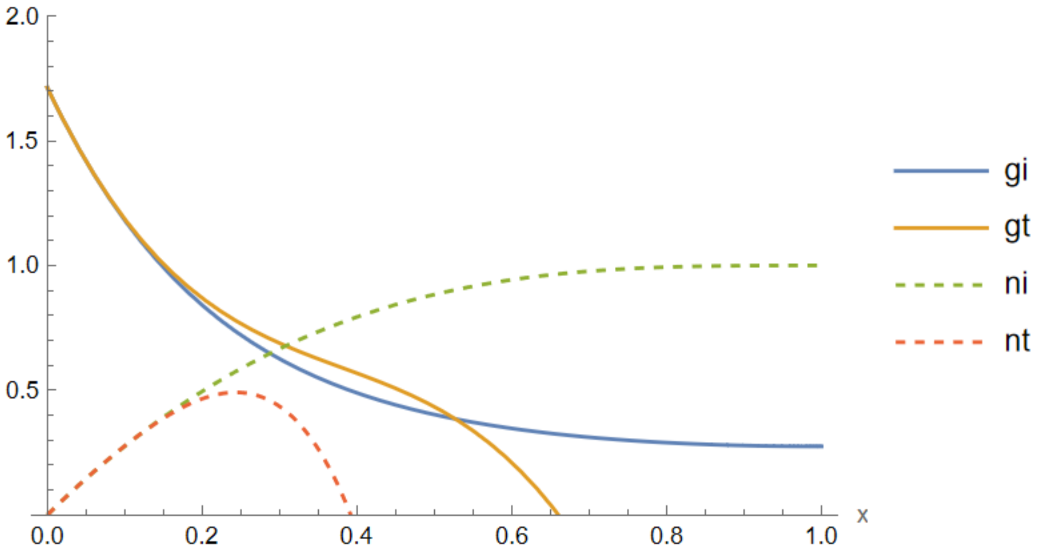
Wormhole examples



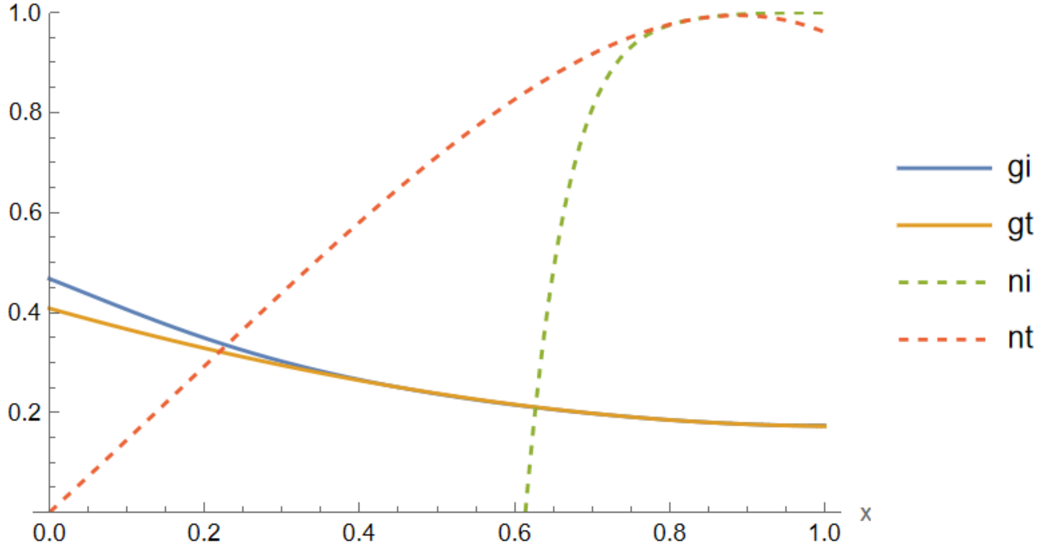
Wormhole examples with beta

- Example 3

Example 4



$\{1.48066\times10^{-17}, \{\beta\to0.501936, aa2\to0.327455, bb3\to-0.680223, r\to0.524004, AA1\to3.06577, BB0\to1.71347, x\to0.000519844\}\}$

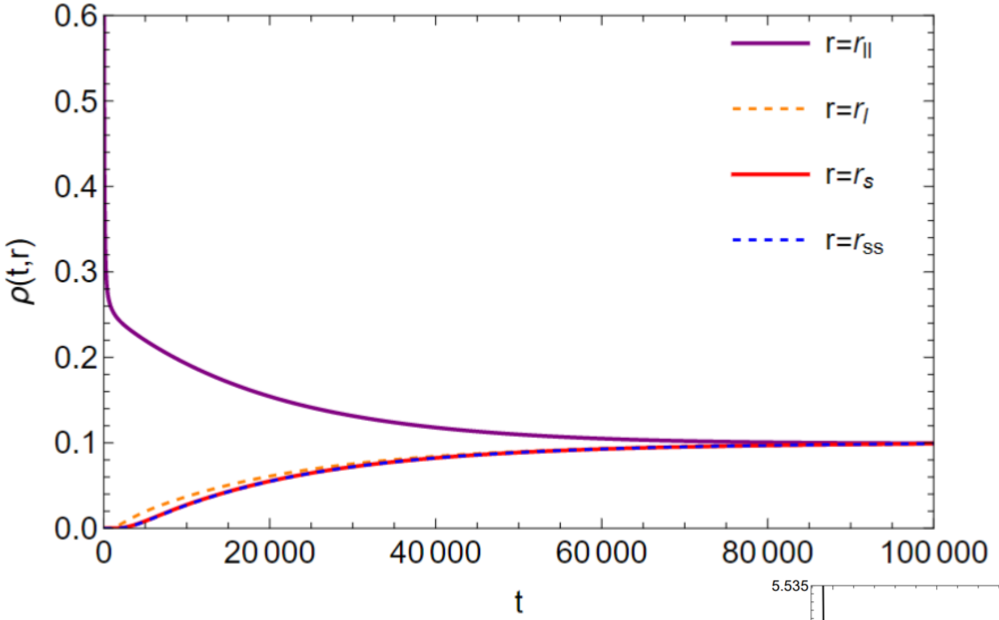


$\{3.2155\times10^{-16}, \{\beta\to0.0706116, aa2\to0.34649, bb3\to-3.53033, r\to0.41619, AA1\to1.43351, BB0\to0.408057, x\to0.83693\}\}$

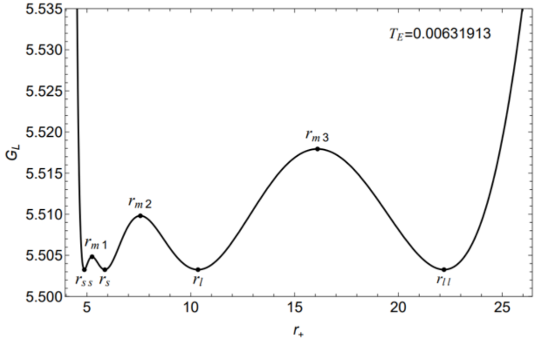
- With beta we can matches the second derivative

Probability evolution for four stable phases at middle temperature

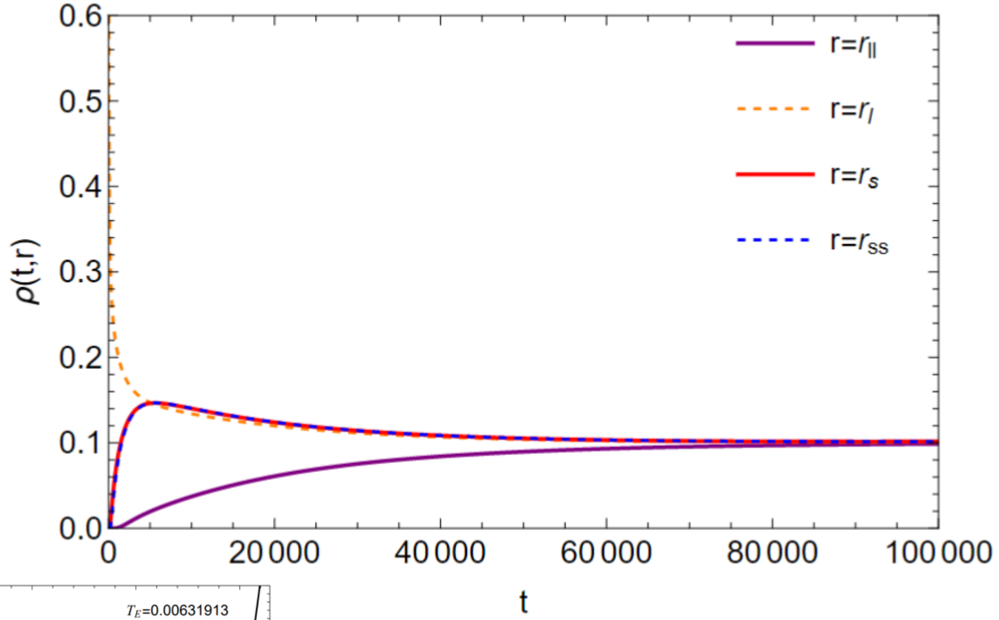
- Initial state is the largest BH



	$t(ll \to l)$	$t(ll \to s)$	$t(ll \to ss)$
T_L	307	439	447
T_I	1593	1817	1827
T_H	3309	3672	3685



Initial state is the large BH

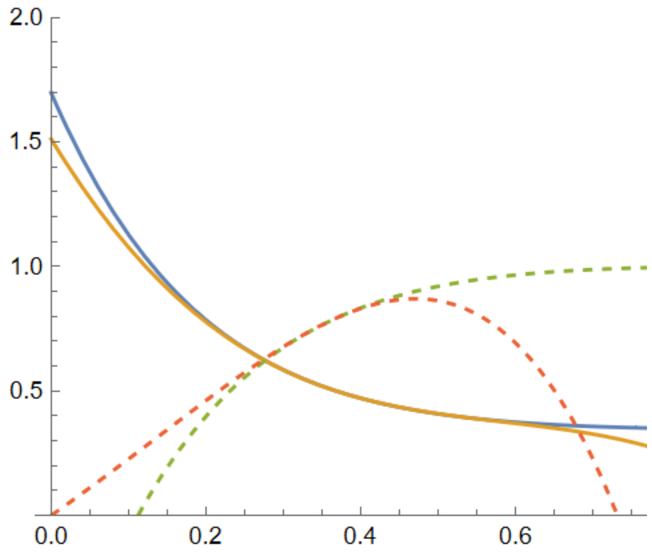


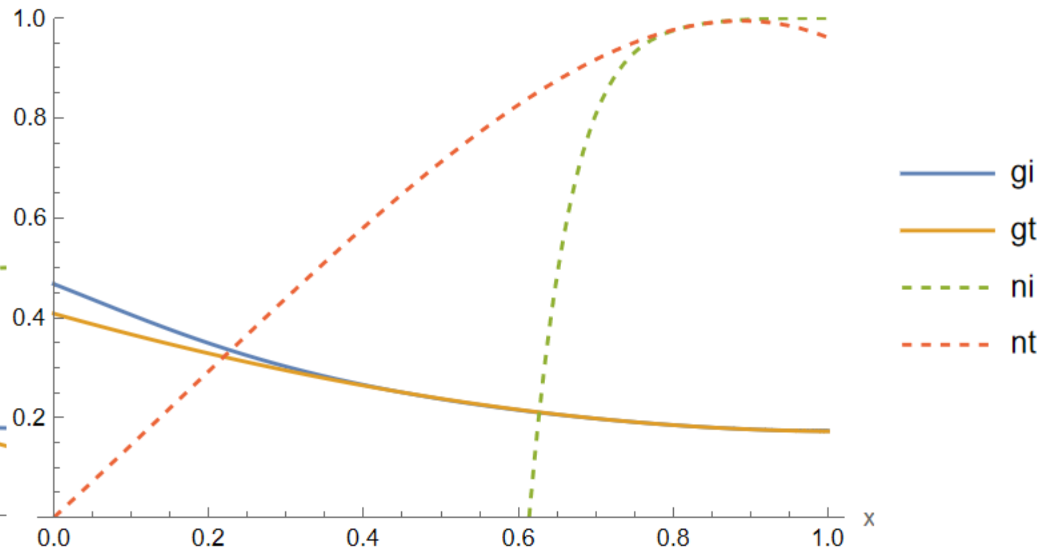
	$t(l \to s)$	$t(l \to ss)$	$t(l \to ll)$
T_L	132	140	1160
T_I	224	234	912
T_H	363	376	466

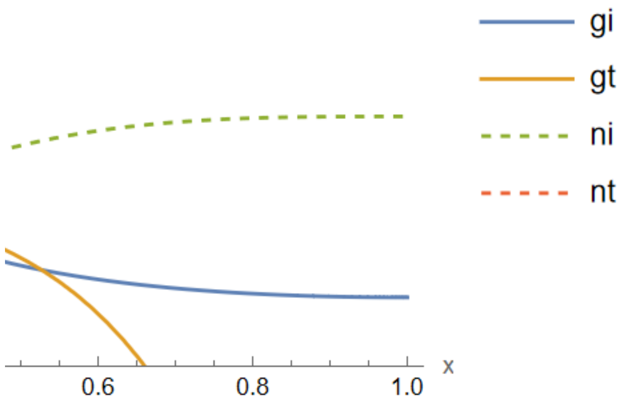
Wormhole examples

- Example 1

Example 2







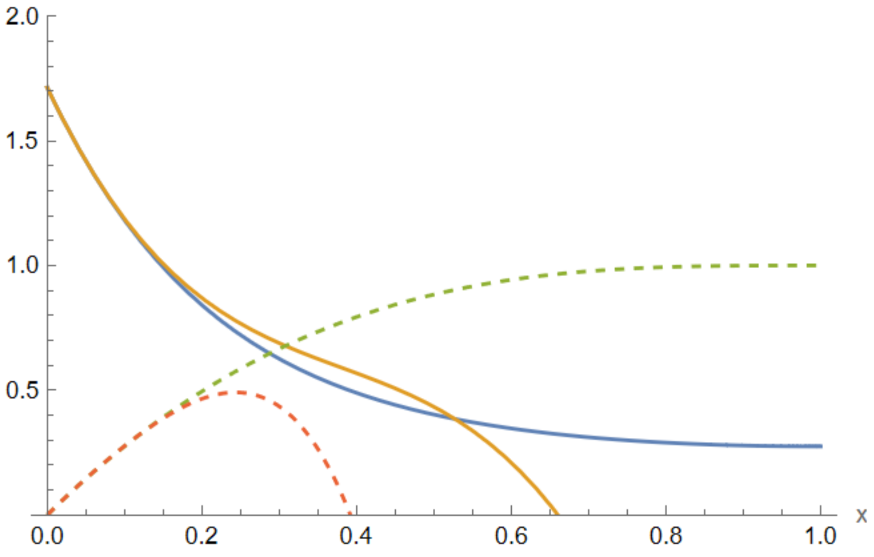
{3.22544 × 10^-15, {β → 0.792546, aa2 → 0.0368723, bb3 → -0.327935,
r → 0.588074, AA1 → 2.1819, BB0 → 1.51153, x → 0.35156}}

{1.48066 × 10^-17, {β → 0.501936, aa2 → 0.327455, bb3 → -0.680223,
r → 0.524004, AA1 → 3.06577, BB0 → 1.71347, x → 0.000519844}}

CHEBYSHEV SPECTRAL METHODS

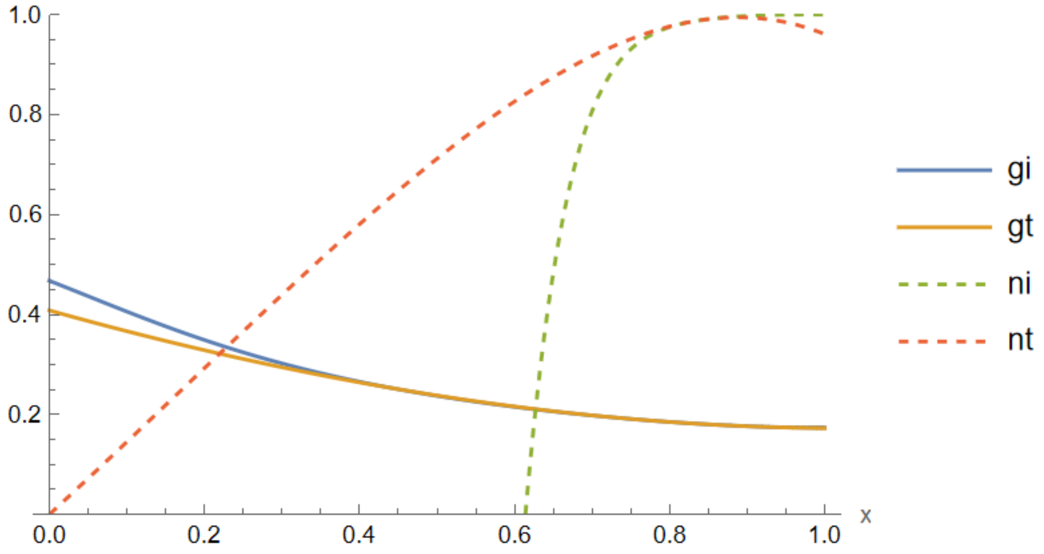
Wormhole examples with beta

- Example 3



$\{1.48066 \times 10^{-17}, \{\beta \to 0.501936, aa2 \to 0.327455, bb3 \to -0.680223, r \to 0.524004, AA1 \to 3.06577, BB0 \to 1.71347, x \to 0.000519844\}\}$

Example 4



$\{3.2155 \times 10^{-16}, \{\beta \to 0.0706116, aa2 \to 0.34649, bb3 \to -3.53033, r \to 0.41619, AA1 \to 1.43351, BB0 \to 0.408057, x \to 0.83693\}\}$

- With beta we can matches the second derivative